

# Solving For Internal Rate Of Return

## Part I - The Bisection Method

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August, 2023

We are given forward rates for an interest rate swap (variable rate leg) and are tasked with calculating the fixed rate equivalent (fixed rate leg). In this white paper we will use the Bisection Method to calculate the fixed rate. To that end we will work through the following hypothetical problem...

### Our Hypothetical Problem

We are given the following interest rate swap parameters given that the swap has a notional value of \$1,000,000 and a term of 10 years with swap payments made at the end of each annual period.

**Table 1: Interest Rate Swap Parameters**

Annual Period	Forward Rate	Discount Factor	Variable Leg Payment	PV Variable Leg Payment	IRR 4.5338%
0	—	1.0000	—	—	-1,000,000.00
1	3.50%	0.9662	35,000.00	33,816.43	35,000.00
2	3.75%	0.9313	37,500.00	34,922.30	37,500.00
3	4.00%	0.8954	40,000.00	35,817.74	40,000.00
4	4.25%	0.8589	42,500.00	36,504.89	42,500.00
5	4.50%	0.8220	45,000.00	36,987.79	45,000.00
6	4.75%	0.7847	47,500.00	37,272.23	47,500.00
7	5.00%	0.7473	50,000.00	37,365.65	50,000.00
8	5.25%	0.7100	52,500.00	37,276.89	52,500.00
9	5.50%	0.6730	55,000.00	37,016.10	55,000.00
10	5.75%	0.6364	1,057,500.00	673,019.98	1,057,500.00
Total	—	—	1,462,500.00	1,000,000.00	462,500.00

**Note:** The IRR in the table above was derived by using the Excel IRR function.

Answer the following questions...

**Question 1:** Graph the net present value of the variable leg swap payments at various discount rates.

**Question 2:** Use the Bisection Method to calculate the internal rate of return (i.e. the fixed rate).

### Building Our Model

We will define the function  $F(r)$  to be the net present value of a time series of cash flows discounted at the annual fixed interest rate  $r$ , the variable  $N$  to be the notional value of the swap, the variable  $C_t$  to be cash flow at the end of time  $t$  (variable leg payment), and the variable  $T$  to be swap term in years. The equation for the net present value of the swap at time zero is...

$$F(r) = \left[ \sum_{t=1}^T C_t \left( 1 + r \right)^{-t} \right] - N \quad (1)$$

We will define the variables  $a$ ,  $b$ , and  $c$  to be interest rates where  $a \leq b \leq c$ . The definitions of these interest rate variables are...

$$\begin{aligned} a &= \text{Discount rate at beginning of rate interval} \\ b &= \text{Discount rate at midpoint of rate interval} \\ c &= \text{Discount rate at end of rate interval} \end{aligned} \quad (2)$$

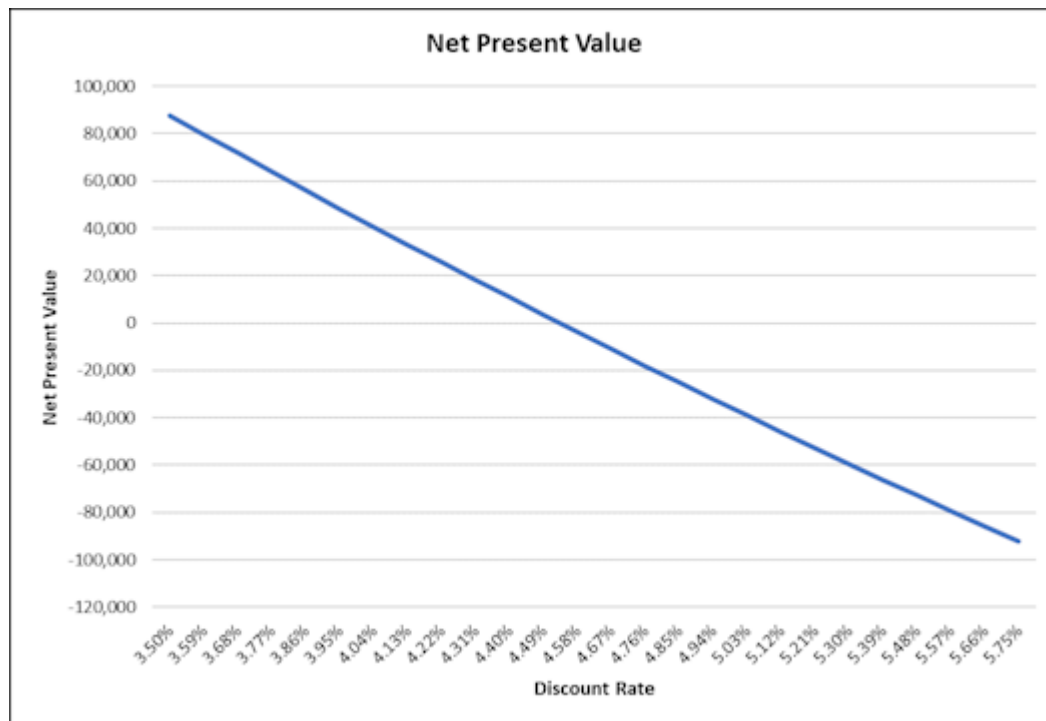
**The Bisection method:** The bisection method is a root-finding method that applies to any continuous function for which one knows two values with opposite signs. The method consists of repeatedly bisecting the interval defined by these values and then selecting the subinterval in which the function changes sign, and therefore must contain a root. It is a very simple and robust method, but it is also relatively slow. The subinterval selection process is as follows...

$$\begin{aligned} \text{Select [a,b]: if... } F(a) > 0 \text{ ...and... } F(b) < 0 \text{ ...then... } \hat{a} = a, \hat{c} = b, \hat{b} &= \frac{\hat{a} + \hat{c}}{2} \\ \text{Select [b,c]: if... } F(b) > 0 \text{ ...and... } F(c) < 0 \text{ ...then... } \hat{a} = b, \hat{c} = c, \hat{b} &= \frac{\hat{a} + \hat{c}}{2} \end{aligned} \quad (3)$$

We iterate the subinterval selection process until Equation (1) is approximated (the right and left side of that equation are close enough).

## The Answers To Our Hypothetical Problem

**Question 1:** Graph the net present value of the variable leg swap payments at various discount rates (NPV Equation (1) above).



Per the graph above, the fixed rate is approximately 4.58% (NPV is approximately zero at that point).

**Question 2:** Use the Bisection Method to calculate the internal rate of return (fixed rate).

We need to determine the start and end interest rates for our iteration process. The start and end rate will be the minimum and maximum, respectively, of the forward rates in Table 1 above. We know that the minimum rate will result in a positive NPV and the maximum rate will result in a negative NPV, which is what we want. The starting rate interval will be...

$$a = 3.5000\%, c = 5.7500\%, \text{ and } b = \frac{3.5000\% + 5.7500\%}{2} = 4.6250\% \quad (4)$$

Using the start rates in Equation (4) above, the iteration process to 25 iterations is...

Iteration	Interest Rate			Net Present Value of Cash Flow			Subinterval Selected
	a	b	c	a	b	c	
0	3.5000%	4.6250%	5.7500%	87,672.66	-7,308.39	-92,334.06	ab
1	3.5000%	4.0625%	4.6250%	87,672.66	38,858.88	-7,308.39	bc
2	4.0625%	4.3438%	4.6250%	38,858.88	15,454.71	-7,308.39	bc
3	4.3438%	4.4844%	4.6250%	15,454.71	3,994.27	-7,308.39	bc
4	4.4844%	4.5547%	4.6250%	3,994.27	-1,676.63	-7,308.39	ab
5	4.4844%	4.5195%	4.5547%	3,994.27	1,153.91	-1,676.63	bc
6	4.5195%	4.5371%	4.5547%	1,153.91	-262.59	-1,676.63	ab
7	4.5195%	4.5283%	4.5371%	1,153.91	445.35	-262.59	bc
8	4.5283%	4.5327%	4.5371%	445.35	91.31	-262.59	bc
9	4.5327%	4.5349%	4.5371%	91.31	-85.66	-262.59	ab
10	4.5327%	4.5338%	4.5349%	91.31	2.82	-85.66	bc
11	4.5338%	4.5344%	4.5349%	2.82	-41.42	-85.66	ab
12	4.5338%	4.5341%	4.5344%	2.82	-19.30	-41.42	ab
13	4.5338%	4.5340%	4.5341%	2.82	-8.24	-19.30	ab
14	4.5338%	4.5339%	4.5340%	2.82	-2.71	-8.24	ab
15	4.5338%	4.5338%	4.5339%	2.82	0.05	-2.71	bc
16	4.5338%	4.5339%	4.5339%	0.05	-1.33	-2.71	ab
17	4.5338%	4.5339%	4.5339%	0.05	-0.64	-1.33	ab
18	4.5338%	4.5339%	4.5339%	0.05	-0.29	-0.64	ab
19	4.5338%	4.5338%	4.5339%	0.05	-0.12	-0.29	ab
20	4.5338%	4.5338%	4.5338%	0.05	-0.03	-0.12	ab
21	4.5338%	4.5338%	4.5338%	0.05	0.01	-0.03	bc
22	4.5338%	4.5338%	4.5338%	0.01	-0.01	-0.03	ab
23	4.5338%	4.5338%	4.5338%	0.01	0.00	-0.01	bc
24	4.5338%	4.5338%	4.5338%	0.00	-0.01	-0.01	ab
25	4.5338%	4.5338%	4.5338%	0.00	0.00	-0.01	ab

**Answer:** At iteration 25 the rate intervals converge to approximately 4.5338% and the NPV intervals converge to near-zero. A fixed rate of 4.5338% is the answer to the question, and is also the Excel IRR calculation in Table 1 above.

Note that the NPV values in the table above are rounded to two decimal points.